

Tomonaga-Luttinger liquid with reservoirs in a multi-terminal geometry.

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We propose a formalism which uses boundary conditions imposed on the Luttinger liquid (LL) to describe the transport properties of a LL coupled to reservoirs. The various boundary conditions completely determine linear transport in the joint system reservoirs+LL. As an illustration we consider an exactly solvable microscopic model in a multi-terminal geometry for which such boundary conditions can be explicitly derived; in this model the Landauer-Büttiker formalism fails: if it were valid, the relation between the conductance matrix elements and the reflection and transmission coefficients could yield negative probabilities. We then apply our formalism to a discussion of shot noise through an impurity in a LL connected to two reservoirs.

I. INTRODUCTION

For Fermi liquids coupled to many reservoir leads linear transport properties can be described by the Landauer-Büttiker formalism whose key idea is to relate transport properties of each single electron at the Fermi level to some transmission and reflection probabilities that characterize the scattering properties of electrons^{1,2}. The Landauer-Büttiker formalism states that the conductance matrix elements G_{np} defined for each conducting channel by $i_n = \sum_p G_{np} V_p$ (where i_n and V_p are respectively the current injected by reservoir n and the voltage of reservoir p) are related to the transmission (T_{np}) and reflection (R_n) probabilities of single electron:

$$G_{nn} = \frac{e^2}{h} (1 - R_n) \quad (1)$$

$$G_{np} = -\frac{e^2}{h} T_{np}. \quad (2)$$

Therefore: (i) $G_{nn} > 0$ and $G_{np} < 0$ and both are bounded by e^2/h ; (ii) in the absence of any backscattering G_{nn} has a finite universal value $G_0 = e^2/h$ which leads in a two-terminal geometry to the so-called universal contact resistance that reflects the (inelastic) relaxation process of each electron that leaves the sample and enters a given reservoir.

For strongly correlated electronic systems such as the Luttinger Liquid (LL) the single particle scattering matrix approach essential to the Landauer-Büttiker formalism becomes inadequate because in a LL the single-particle Green function has no quasiparticle pole which means that electrons do not propagate³.

Nevertheless, the Landauer-Büttiker formalism has been widely used to interpret recent two and multi-terminal linear transport measurements⁴⁻⁶ on *physical systems that are believed to be good examples of LL*^{7,8}. More precisely several experiments on quantum wires fabricated by cleaved edge overgrowth found two-terminal conductance plateaux at nonuniversal values: $G = 0.8, 0.85, 0.9^5$ and as low as 0.45 in units of the quantum of conductance G_0 (per channel)⁷. In Ref.⁵ results for a three terminal geometry are also reported and

show that $R_n \neq 0$ if interpreted in the Landauer-Büttiker frame. Assuming then that electrons in the wire are non-interacting these values can be understood by invoking some backscattering due to the coupling of the ballistic wire to the two-dimensional reservoirs⁵. Similarly individual single wall carbon nanotubes were found with either a two-terminal conductance $G = G_0^6$ but also with $G \sim 0.5 - 0.6 G_0^9$.

If one believes that quantum wires and carbon nanotubes constitute realizations of the LL the interpretation of these non universal ohmic conductance plateaux poses a serious problem in two respects: (i) if the smaller conductances are interpreted as resulting from some backscattering (in agreement with a Landauer-Büttiker interpretation) then this is contradictory to the expectation that in a LL backscattering should lead to power law (non-ohmic) corrections; (ii) moreover in the framework of the LL theory several earlier theoretical papers found that the two-terminal conductance of a LL should be unaffected by the interactions and stick to a universal value G_0 yielding the same contact resistance as in non-interacting systems¹⁰.

Actually a theoretical analysis of the transport properties of a LL in a multi-terminal geometry has not really been developed.

In this paper we develop a formalism to describe the linear transport properties of a LL coupled to an arbitrary number of electrodes, which play the role of charge reservoirs for the LL. Instead of a full-fledged microscopic approach which is obviously unassailable we propose a formalism in which the coupling of a LL to reservoirs is taken into account by considering a LL subjected to boundary conditions. Our approach generalizes several earlier papers which take a similar stand for the modelling of a coupled system¹¹⁻¹⁵.

Our basic idea is as follows: (i) we observe that in a LL a chiral decoupling occurs, i.e. each chirality is independent of the other and is responsible for the transport of current in one direction, and therefore the transport properties are completely specified by the associated chiral chemical potentials. (ii) Imposing linear relations to the terminal voltages determines the values of these chi-

ral potentials: each set of relation can therefore be seen as a set of boundary conditions imposed on the LL. We consider that each set of boundary conditions characterizes some kind of coupling of the electrodes to the LL. In other words *as far as the transport properties are concerned* we view a given set of boundary conditions as corresponding to a universality class for the total system consisting of both the LL and the electrodes. That formalism is explained in **section 2**.

To substantiate our views we consider in **section 3** an exactly solvable model of strongly correlated fermions coupled to an arbitrary number of terminals and *for which such boundary conditions can be derived explicitly*. We find that in the toy model of **section 3** the signs of the conductance matrix elements are not fixed: if we were to define reflection and transmission coefficients R_n and T_{np} through equations (1,2) we would get negative probabilities. That the probabilistic view of transport given by the Landauer-Büttiker formalism fails is hardly surprising: transport in a LL is not ensured by electrons (or Landau quasiparticles) but by fractional excitations akin to those of the Fractional Quantum Hall Effect (FQHE)¹⁶.

In **section 4** we specialize to the two-terminal geometry. We show that the boundary conditions are equivalent in that geometry to specifying contact resistances at the source and the drain. We explain how our formalism allows us to classify earlier theories: theories yield distinct conductances because they belong to different universality classes and correspond to distinct boundary conditions. We then consider a LL with an impurity which provides some backscattering within the system but not at the two contacts. We then discuss implications of our formalism in **section 5** for the various theories of shot noise through a single impurity in a LL.

II. BOUNDARY CONDITIONS IN A MULTI-TERMINAL GEOMETRY.

A. Chiral chemical potentials for the LL.

We consider the standard LL hamiltonian density:

$$H = \hbar \frac{u}{2} \left[K^{-1} (\partial_x \Phi)^2 + K \Pi^2 \right] \quad (3)$$

where we have introduced the standard phase field Φ related to the electron density by: $\delta \rho(x, t) = \partial_x \Phi / \sqrt{\pi}$, and its conjugate canonical momentum Π . We also introduce the chiral densities:

$$\rho_{\pm} = \frac{1}{2\sqrt{\pi}} (\partial_x \Phi \pm K \Pi). \quad (4)$$

It is easy to check that these densities are indeed chiral and to show that they are related to the densities of electrons at the right and left Fermi points by:

$$\begin{pmatrix} \rho_+ \\ \rho_- \end{pmatrix} = Q[K] \begin{pmatrix} \rho_+^0 \\ \rho_-^0 \end{pmatrix}, \quad (5)$$

where:

$$Q[a] = \frac{1}{2} \begin{pmatrix} 1+a & 1-a \\ 1-a & 1+a \end{pmatrix}. \quad (6)$$

In terms of these chiral variables the LL hamiltonian is completely decoupled into two chiral hamiltonians. Current will then be induced in the LL by adding conjugate variables to H :

$$\delta H = -\mu_+ \rho_+ - \mu_- \rho_- \quad (7)$$

which defines chiral chemical potentials; as usual when defining chemical potentials they correspond to the energy needed for the creation of a unit particle of a given chirality *within the LL* ($Q_{\pm} = \int dx \rho_{\pm} = 1$). They are not defined as the reservoir chemical potentials. Minimization of the hamiltonian leads easily to the relation:

$$I = \frac{Ke}{h} (\mu_+ - \mu_-). \quad (8)$$

Therefore the two-terminal conductance of a LL as measured against these chiral chemical potentials is:

$$G_K = \frac{Ie}{\mu_+ - \mu_-} = K \frac{e^2}{h}. \quad (9)$$

It is crucial to realize that the experimentally measured conductance $G_{2t} = \frac{Ie}{\mu_S - \mu_D}$ and G_K need not be identical. The chiral chemical potentials (associated with the quasiparticles driving the current in the LL) will indeed usually differ from the reservoir chemical potentials.

B. Boundary conditions.

We now consider N terminals at respective voltages V_n . The geometry is indifferent and is either that of a loop or that of a rod (see Figures 1 and 2). Due to current injection at each terminal the chiral chemical potentials may change their values across them: accordingly we add indices so that μ_n^+ and μ_n^- are on the left of terminal n and change to μ_{n+1}^+ and μ_{n+1}^- on the right of the same terminal. Depending on the geometry for N terminals there will $2N$ or $2N - 2$ chiral chemical potentials.

$$\mu_n^+ = \sum_p a_n^p e V_p \quad (10)$$

$$\mu_n^- = \sum_p b_n^p e V_p \quad (11)$$

The Fermi energy of the LL is taken as the origin of the voltages. These equations simply express an equilibrium condition for the chemical potentials of the LL with those of the reservoirs. Our point of view is therefore in a sense a thermodynamical one.

The joint system (LL+electrodes) is therefore reduced to an isolated LL (i.e. with no electrodes) on which these boundary conditions are imposed. The advantage of such an approach is that it avoids the complications of microscopics. To borrow the language of the renormalization group we view the boundary conditions as characterizing universality classes of the system (LL+electrodes). Any set of boundary conditions labels a fixed point of the joint system and completely determines linear transport as will be shown below.

Since eV_p is the chemical potential of reservoir p charge conservation results trivially in:

$$1 = \sum_p a_n^p = \sum_p b_n^p. \quad (12)$$

The current injected at each terminal is simply the difference between the current circulating on the right of the terminal and the current on the left of it:

$$i_n = K \frac{e}{h} [(\mu_{n+1}^+ - \mu_{n+1}^-) - (\mu_n^+ - \mu_n^-)]. \quad (13)$$

Taking into account the boundary conditions eq.(10) leads to:

$$G_{nn} = K \frac{e^2}{h} (a_{n+1}^n - a_n^n + b_n^n - b_{n+1}^n) \quad (14)$$

$$G_{np} = K \frac{e^2}{h} (a_{n+1}^p - a_n^p + b_n^p - b_{n+1}^p). \quad (15)$$

It is trivial to check that the following identities are obeyed:

$$\sum_p G_{np} = 0 \quad (16)$$

$$\sum_p G_{pn} = 0. \quad (17)$$

The first identity implements the fact that the origin of potentials is arbitrary (gauge invariance); the second identity is just current conservation.

The requirement that Onsager-Casimir relations and positive semidefiniteness of the conductance matrix (this ensures dissipation of energy) be realized evidently puts further constraints on the coefficients a_n^p and b_n^p (such as positivity of the diagonal elements G_{nn}) but we will not use them in this paper.

These boundary conditions can then be straightforwardly generalized to the case of several conducting channels as in carbon nanotubes.

III. A MULTI-TERMINAL MODEL.

We introduce in this part a model derived from one initially proposed by Chamon and Fradkin for the FQHE¹⁷. The model consists in a chiral Luttinger liquid in contact with N reservoirs through point contacts in the strong coupling limit (no backscattering at the contact) (see Figure 3). We will show that our boundary conditions

formalism can be explicitly derived for this model of strongly correlated electrons; moreover we show that the conductance matrix elements can surprisingly have signs forbidden in the Landauer-Büttiker formalism.

A. Some useful preliminaries.

We review first relevant results for the lagrangian of a chiral LL coupled to a single reservoir¹⁷:

$$L = L_{edge} + L_{reservoir} + L_{tunnel} \quad (18)$$

$$L_{edge} = \frac{1}{4\pi} \partial_x \varphi (\partial_t - u \partial_x \varphi) \quad (19)$$

$$L_{tunnel} = \Gamma \delta(x) e^{i \frac{e \Delta V t}{h}} \Psi_{edge}^+(x, t) \Psi_{reservoir} + h.c. \quad (20)$$

The lagrangian for the reservoir is taken to be that of free chiral electrons: the electrons are chiral because free electrons on a half-line are tantamount to free chiral electrons on a full line. $\Delta V = V_{reservoir} - V_{edge}^{in}$ is the potential difference between the reservoir and the incoming edge electrons. The edge electron operator is: $\Psi_{edge} = e^{-i \frac{1}{\sqrt{\nu}} \varphi}$ where $\nu = \frac{1}{2n+1}$ is the Quantum Hall state filling factor.

In the strong coupling limit, the tunneling current can then be shown to be:

$$i_{tunnel} = \frac{2\nu}{\nu+1} \frac{e^2}{h} (V_{reservoir} - V_{edge}^{in}) \quad (21)$$

which can also be expressed in terms of the chemical potential V_{edge}^{out} after the tunneling event:

$$i_{tunnel} = \nu \frac{e^2}{h} (V_{edge}^{out} - V_{edge}^{in}). \quad (22)$$

That expression for the tunneling current is valid at the strong coupling fixed point ($\Gamma = \infty$): therefore transmission is always perfect with no backscattering at the contact. Away from the fixed point (at finite Γ), there would be a contribution due to backscattering which vanishes as $\frac{1}{\Gamma^K} \Delta V^{2(K-1)}$. *In our model we will also work in this ohmic limit of no backscattering.*

Chamon and Fradkin then consider N_R such tunnel point contacts, each at the same potential $V_{reservoir} = V_D$ and then N_L similar point contacts at the potential $V_{reservoir} = V_S$. In this way, they modelize a quantum Hall bar at filling ν with two 2D reservoirs on the left and at the right of the sample, which are connected to the Hall bar through respectively N_L and N_R point contacts. An assumption underlying this model is therefore that the point contacts are incoherent, i.e. each tunneling at each point contact can be considered independently of the other point contacts. The two-terminal conductance can then be easily extracted and depends on the number of contacts N_R and N_L ; in particular the conductance differs in general from $G_0 = \frac{e^2}{h}$:

$$G(N_L, N_R) = \nu \frac{e^2}{h} \frac{\left[1 - \left(\frac{\nu-1}{\nu+1}\right)^{N_L}\right] \left[1 - \left(\frac{\nu-1}{\nu+1}\right)^{N_R}\right]}{\left[1 - \left(\frac{\nu-1}{\nu+1}\right)^{N_R+N_L}\right]}. \quad (23)$$

B. A toy model.

We turn now to our model: it consists in a chiral Luttinger liquid with Luttinger parameter $\nu = 1/(2n+1)$ in a circle geometry with N terminals (Figure 3). That problem is easily solved by observing that it is a generalization of Chamon and Fradkin model found by allowing each of N point contacts to have distinct potentials. The n -th terminal has potential V_n , and due to the current i_n which tunnels through it the chemical potential of the chiral LL is raised from μ_n to μ_{n+1} . The associated tunnel lagrangian L_{tunnel}^n is exactly similar to L_{tunnel} given above in eq.(18), except for with the location of the tunneling center.

Let us focus on the n -th terminal; using equations (21,22); the current i_n is:

$$i_n = \frac{2\nu}{\nu+1} \frac{e}{h} (eV_n - \mu_n), \quad (24)$$

$$= \nu \frac{e}{h} (\mu_{n+1} - \mu_n). \quad (25)$$

This yields the relation:

$$\mu_{n+1} = \alpha \mu_n + (1 - \alpha) e V_n \quad (26)$$

where $\alpha = \frac{\nu-1}{\nu+1}$. Since we work on a circle with N terminals: $\mu_{N+1} = \mu_1$; we then deduce that each potential μ_n can be completely expressed in terms of the potentials of all the terminals:

$$\mu_n = \frac{(1-\alpha)\alpha^{n-1}}{1-\alpha^N} \left[\sum_{p=1}^{n-1} \frac{eV_p}{\alpha^p} + \alpha^N \sum_{p=n}^N \frac{eV_p}{\alpha^p} \right]. \quad (27)$$

According to eq. (25), the current injected by the n -th terminal $i_n = G_{nn}V_n + \sum_{p \neq n} G_{np}V_p$ yields the conductance matrix:

$$G_{nn} = \frac{e^2}{h} \frac{(1+\alpha)(1-\alpha^{N-1})}{1-\alpha^N} \quad (28)$$

$$G_{np} = -\frac{e^2}{h} \frac{(1-\alpha^2)\alpha^{d(n,p)-1}}{1-\alpha^N}, n \neq p \quad (29)$$

where $d(n,p) = n-p$ for $n > p$, or $d(n,p) = N+n-p$ for $n < p$.

Several points are quite noteworthy:

(1) The boundary conditions described in **section 2** are implemented exactly by eq.(27) with the coefficients:

$$a_n^p = \frac{(1-\alpha)\alpha^{d(n,p)-1}}{1-\alpha^N}, n \neq p \quad (30)$$

$$a_n^n = \frac{(1-\alpha)\alpha^{N-1}}{1-\alpha^N}, \quad (31)$$

(2) In the scattering approach the conductance matrix is directly related to probabilities of reflection and transmission through: $G_{nn} = \frac{e^2}{h} (1 - R_n)$ and $G_{np} = -\frac{e^2}{h} T_{np}$. Therefore G_{nn} is a positive number while G_{np} is negative. However in our model G_{np} can be *positive* depending on the value of $d(n,p)$. More precisely $G_{n,p}$ and $G_{n,p+1}$ have alternating signs: this means that even if electrodes p and $p+1$ have the same potential currents issued from them are flowing in opposite directions to electrode n . This would have been impossible for non-interacting electrons.

(3) Current conservation and gauge invariance are implemented since: $\sum_p G_{pn} = 0 = \sum_p G_{np}$.

(4) The quantity $\sum_{n,p} V_n V_p G_{np}$ is always positive, which ensures dissipation of energy.

Proof: It is equivalent to show that $\sum_{n,p} V_n V_p \frac{1}{2}(G_{np} + G_{pn}) \geq 0$. It is enough for that purpose to show that the eigenvalues of the matrix $\frac{1}{2}(G_{np} + G_{pn})$ are all positive. But $\frac{1}{2}(G_{np} + G_{pn})$ is a circulant matrix, i.e. a square matrix whose rows are obtained by displacing the matrix elements of the first row by one column. For a circulant matrix whose first row is (a_1, \dots, a_N) the $k = 1, \dots, N$ eigenvalues are equal to $P(r_k)$ where $r_k = \exp \frac{i2\pi k}{N}$ is one of the N th roots of unity and the polynomial $P(X) = \sum_{p=0}^{N-1} a_{p+1} X^p$. For the matrix $\frac{1}{2}(G_{np} + G_{pn})$ the first row (a_1, \dots, a_N) is given by:

$$a_1 = G_{11} = \frac{e^2}{h} \frac{(1+\alpha)(1-\alpha^{N-1})}{1-\alpha^N},$$

$$a_i = \frac{1}{2}(G_{1i} + G_{i1}) = -\frac{e^2}{h} \frac{(1-\alpha^2)}{1-\alpha^N} \left(\frac{\alpha^{i-2} + \alpha^{N-i}}{2} \right), i > 1.$$

Therefore the eigenvalues are given by:

$$P(r_k) = \frac{(1+\alpha)^2(1 - \cos \frac{2\pi k}{N})}{(1+\alpha^2 - 2\alpha \cos \frac{2\pi k}{N})} \geq 0.$$

QED.

One eigenvalue vanishes ($P(r_0) = 0$); it corresponds to the eigenvector $(V_1, \dots, V_N) = (1, \dots, 1)$ which implements gauge invariance (since the origin of voltages is arbitrary no current can flow if all voltages are equal).

(5) It is easy to check that Onsager-Casimir reciprocity relations are obeyed in this model: $G_{np}(\phi = \nu\phi_0) = G_{pn}(\phi = -\nu\phi_0)$. Since a magnetic field is present, under time-reversal one must reverse its sign, which implies that α is changed into $1/\alpha$. Onsager-Casimir reciprocity then follows immediately from eq. (28-29).

(6) An interesting test-case of our model would be an experimental setup with a FQHE disk for which the number of terminals can be changed easily; what we envision is at first an experimental configuration with of course few quantum point contacts (at least three in order to observe the alternation of signs of non-diagonal elements of the conductance matrix), which are tuned through a gate voltage, so that the point contacts may be added or removed at will.

IV. BOUNDARY CONDITIONS FOR A LL IN A TWO-TERMINAL GEOMETRY.

A. Boundary conditions

As an illustration of our formalism we consider the simplest case of a two-terminal geometry with a source and a drain at voltages V_S and V_D (see Figure 4).

We take as a boundary condition the relation:

$$\begin{pmatrix} \mu_+ \\ \mu_- \end{pmatrix} = A \begin{pmatrix} eV_S \\ eV_D \end{pmatrix} \quad (32)$$

where the matrix A is:

$$A = \begin{pmatrix} a^S & a^D \\ b^S & b^D \end{pmatrix}$$

and where conservation of the number of particles imposes $a^S + a^D = 1 = b^S + b^D$. The parameter space is therefore two-dimensional. The conductance matrix therefore is:

$$\begin{pmatrix} G_{SS} & G_{SD} \\ G_{DS} & G_{DD} \end{pmatrix} = K \frac{e^2}{h} (a^S - b^S) \begin{pmatrix} 1 & -1 \\ -1 & 1 \end{pmatrix}.$$

Therefore the two-terminal conductance $G_{2t} = \frac{I}{V_S - V_D} = \frac{I_S}{V_S - V_D}$ is:

$$G_{2t} = G_{SS} = K \frac{e^2}{h} (a^S - b^S) = G_K \det A \quad (33)$$

where the intrinsic conductance of the LL is $G_K = \frac{Ie}{\mu_+ - \mu_-} = K \frac{e^2}{h}$ (see **section 2**). Since the two-terminal conductance depends on the difference $(a^S - b^S) = \det A$ distinct boundary conditions or distinct coupling between the reservoirs and the LL can lead to the same two-terminal conductance value. In contrast a given set of the boundary conditions specifies unambiguously the conductance value. If the two terminals are coupled in a symmetric manner to the LL one has a further condition: $a^S = b^D$. The parameter space is then one-dimensional and $G = K \frac{e^2}{h} (2a^S - 1)$.

B. Contact resistances.

The LL has mean chemical potential $\bar{\mu} = \frac{\mu_+ + \mu_-}{2}$; but the reservoirs have potentials eV_S and eV_D . Therefore there is a discontinuity between the chemical potentials of the reservoirs and the LL. In the standard Landauer-Buttiker picture of the contact resistance, the latter results precisely from such a discontinuity at the boundaries of each reservoir¹⁸: on a length equal to the inelastic scattering length of each reservoir collisions bring back the energy of each particle coming from the metal to that of the reservoir. In our case, it follows immediately from the boundary conditions that:

$$eV_S - \bar{\mu} = \frac{2 - a^S - b^S}{2} (eV_S - eV_D) = \frac{h}{2Ke} \frac{2 - a^S - b^S}{(a^S - b^S)} I \quad (34)$$

$$\bar{\mu} - eV_D = \frac{a^S + b^S}{2} (eV_S - eV_D) = \frac{h}{2Ke} \frac{a^S + b^S}{(a^S - b^S)} I$$

which shows that there are two contact resistances:

$$R_S = \frac{R_K}{2} \frac{2 - a^S - b^S}{(a^S - b^S)}$$

$$R_D = \frac{R_K}{2} \frac{(a^S + b^S)}{(a^S - b^S)}$$

where the intrinsic resistance of the LL is simply: $R_K = 1/G_K$. These expressions also show that the two terminal conductance is obtained from a series addition law of the two contact resistances: $R_S + R_D = \frac{h}{Ke^2} \frac{1}{(a^S - b^S)} = R_{2t} (= \frac{1}{G_{2t}})$. This implies that our boundary conditions incorporate an assumption of *incoherence between the contacts*. The above two equalities eq.(34) are completely equivalent to the boundary conditions. The two degrees of freedom in the boundary conditions simply reflect the fact that there are two contact resistances. In the two terminal geometry we may therefore rewrite the boundary conditions matrix A in terms of the contact resistances:

$$A = \frac{1}{(R_S + R_D)} \begin{pmatrix} R_K^c + R_D & -R_K^c + R_S \\ -R_K^c + R_D & R_K^c + R_S \end{pmatrix}, \quad (35)$$

where we have defined an intrinsic contact resistance as: $R_K^c = \frac{1}{2G_K} = \frac{1}{2KG_0}$.

Rewriting eq.(34) in terms of the chemical potentials leads to an expression equivalent to the boundary condition expressed by eq.(35):

$$eV_S - \bar{\mu} = R_S I = R_S G_{2t} (eV_S - eV_D) \quad (36)$$

$$\bar{\mu} - eV_D = R_D I = R_D G_{2t} (eV_S - eV_D)$$

C. Chamon and Fradkin model.

In the Chamon-Fradkin model a Hall bar has two terminals on its left and on its right¹⁷. There are N_R (resp. N_L) point contacts at the right and left terminals. On the upper and lower edges there are chiral Luttinger liquids flowing in opposite directions. However the *sum of the chiral hamiltonians for each chiral edge is exactly identical to that of a non-chiral LL with parameter $K = \nu$* . Through our phenomenological formalism, this allows us to describe Chamon and Fradkin microscopic model of a chiral LL as a non-chiral LL but with peculiar boundary conditions. If we make the reasonable assumption that the contact resistances R_S and R_D depend on N_L and N_R respectively (and not on both N_L and N_R), there is a single boundary condition corresponding to Chamon and Fradkin model. Given the two-terminal conductance in eq.(23) we find the boundary conditions in eq.(35) with:

$$\frac{1}{2R_D} = \nu \frac{e^2}{h} \frac{\left(1 - \left(\frac{\nu-1}{\nu+1}\right)^{N_R}\right)^2}{\left(1 - \left(\frac{\nu-1}{\nu+1}\right)^{2N_R}\right)} \quad (37)$$

$$\frac{1}{2R_S} = \nu \frac{e^2}{h} \frac{\left(1 - \left(\frac{\nu-1}{\nu+1}\right)^{N_L}\right)^2}{\left(1 - \left(\frac{\nu-1}{\nu+1}\right)^{2N_L}\right)} \quad (38)$$

This proves that for a non-chiral LL connected to two reservoirs it is perfectly possible theoretically to have non-trivial contact resistances (different from $\frac{1}{2G_0}$). The contact resistance $\frac{1}{2G_0}$ is retrieved for $N_R = N_L = 1$.

D. Comparison with earlier theories.

Let us now discuss earlier theories of the two-terminal conductance and show that the differences between their predicted values of G_{2t} can be completely understood within our boundary condition formalism. The differences stem from the hypotheses these theories make, implying different boundary conditions and therefore different universality classes for the joint system LL+electrodes.

1. Boundary conditions corresponding to $G_{2t} = Ke^2/h$:

Initially the conductance of the LL was thought to be $G_{2t} = Ke^2/h$ following the response function calculation³. Such calculation implicitly assumes an inversion symmetry between source and drain. Comparing this conductance value with our computations shows that $R_S = R_D = R_K^c$. Going back to the boundary conditions equations yields the universality class:

$$\begin{pmatrix} \mu_+ \\ \mu_- \end{pmatrix} = \begin{pmatrix} eV_S \\ eV_D \end{pmatrix}. \quad (39)$$

This allows a physical interpretation of the two-terminal conductance $G_{2t} = Ke^2/h$ for symmetric electrodes: the conductance will be equal to the Luttinger liquid parameter whenever there is an equilibrium between a given reservoir and one of the two chiralities within the LL. In other words the current injected by each electrode is completely chiral. Such boundary conditions are realized in the FQHE: one finds indeed a two-terminal conductance $G_{2t} = Ke^2/h$ for a filling fraction K which corresponds to a LL with parameter K . It is also easy to show that eq.(39) is realized in the FQHE for chiral edges. Indeed the chiral currents $i_{\pm} = K \frac{e}{h} \mu_{\pm}$ can be shown by using linear response to be also equal to $i_{\pm} = K \frac{e}{h} \mu_{S/D}$ which then implies immediately eq.(39). Such a conductance can also be recovered for a non-chiral LL by using

a Kubo formula where it is assumed that an external field E_0 applied on a length L creates a voltage drop $-E_0L = eV_{SD} = eV_S - eV_D = \mu_+ - \mu_-$. In our formalism the specification of the external electrical field as $E_0 = -\frac{\mu_+ - \mu_-}{L}$ is then understood as implying an equilibrium between the reservoirs and a chirality of the LL. But more generally this needn't be the case and a more general linear relation between E_0 and $\mu_+ - \mu_-$ might hold, leading to another value of the two-terminal conductance.

As is clear from our formalism in absence of symmetry between source and drain the two terminal conductance value $G_{2t} = Ke^2/h$ can be obtained from any boundary conditions such that $R_S + R_D = 2R_K^c$.

2. Boundary conditions corresponding to $G_{2t} = e^2/h$:

As we have already mentioned in the introduction many other theoretical approaches predict a non-renormalized conductance value $G_{2t} = G_0 = e^2/h$ per channel (these calculations also implicitly assume a mirror symmetry between source and drain so that $R_S = R_D$). This shows that $R_S = R_D = \frac{1}{2G_0}$ so that the corresponding boundary conditions can be written in term of the matrix $Q[x]$ defined in eq.(6) of **section 2**:

$$\begin{pmatrix} \mu_+ \\ \mu_- \end{pmatrix} = Q[K^{-1}] \begin{pmatrix} eV_S \\ eV_D \end{pmatrix}. \quad (40)$$

or

$$\begin{pmatrix} eV_S \\ eV_D \end{pmatrix} = Q[K] \begin{pmatrix} \mu_+ \\ \mu_- \end{pmatrix}. \quad (41)$$

Besides for the inhomogeneous LL model (a LL for which the LL parameter $K(x)$ varies with position, $K(0 < x < L) = K$ and $K(x) = 1$ otherwise) by using continuity equations for the phase fields across the boundaries it can be explicitly shown that²:

$$\begin{pmatrix} eV_S \\ eV_D \end{pmatrix} = \begin{pmatrix} \mu_+^0 \\ \mu_-^0 \end{pmatrix}, \quad (42)$$

where $\mu_{\pm}^0 = \frac{\partial H}{\partial N_{\pm}^0}$ are the chiral potentials for non-interacting electrons, that is if $K = 1$ (N_{\pm}^0 is the number of fermions at the right or left Fermi points). But this can be reexpressed in terms of the chiral chemical potentials by noting that the chiral densities are related to the left or right moving fermions densities by eq.(5), which results in

$$\begin{pmatrix} \mu_+^0 \\ \mu_-^0 \end{pmatrix} = Q[K] \begin{pmatrix} \mu_+ \\ \mu_- \end{pmatrix}. \quad (43)$$

Using the eq.(42,43) leads immediately to our boundary conditions eq.(41).

For completeness, let us mention several papers which have already developed a narrower boundary conditions

point of view to describe linear transport of LL in a two-terminal geometry. Frhlich et al. implicitly considered the boundary conditions $\begin{pmatrix} eV_S \\ eV_D \end{pmatrix} = \begin{pmatrix} \mu_+^0 \\ \mu_-^0 \end{pmatrix}$ or $\begin{pmatrix} eV_S \\ eV_D \end{pmatrix} = \begin{pmatrix} \mu_+ \\ \mu_- \end{pmatrix}$ for a LL and for a chiral LL for the FQHE in reference¹¹; these are subcases of our own formalism. Egger et al. then discussed so-called radiative boundary conditions for the LL¹², and Büttiker et al. derived a similar boundary condition later¹³. Safi showed finally in¹⁴ that the boundary conditions of^{12,13} are all equivalent to $\begin{pmatrix} eV_S \\ eV_D \end{pmatrix} = \begin{pmatrix} \mu_+^0 \\ \mu_-^0 \end{pmatrix}$. Therefore all these approaches boil down within our formalism to choosing one particular boundary condition.

3. Other boundary conditions?

Within our formalism, earlier theories correspond to two particular boundary conditions leading either to $G = Ke^2/h$ or to $G = e^2/h$. But as discussed in the introduction experimental evidence on carbon nanotubes and quantum wires yield conductance plateaux at values which differ from $G = G_0$ per channel^{4,7,5,9}.

The case of quantum wires fabricated by the cleaved edge overgrowth technique is quite noteworthy: on the one hand evidence points towards a LL physics⁷; on the other hand Landauer scattering approach is used to interpret conductance measurements in⁵. Yet Landauer-Büttiker formalism is invalid in that context of strongly correlated electrons! Our formalism resolves the tension because it includes a theory of linear transport which is independent of Landauer scattering approach, while being applicable to the LL.

Whether more general boundary conditions than those implicit in earlier theories are realized must be settled by experiments. But we observe firstly that the Chamon and Fradkin model for two chiral edges can be interpreted as a non-chiral LL connected (albeit in a very peculiar manner) to two large reservoirs; this model then yields a conductance $G \neq G_0$: this implies that at least theoretically there is no grounds for a no-go theorem preventing two-terminal conductance plateaux at values distinct from the quantum of conductance. Secondly it is noteworthy that in the inhomogeneous LL model which falls into the class of boundary condition $\begin{pmatrix} eV_S \\ eV_D \end{pmatrix} = \begin{pmatrix} \mu_+^0 \\ \mu_-^0 \end{pmatrix}$ *injection of current from the reservoirs to the LL is done through a single point contact*. But experimentally both quantum wires and carbon nanotubes have a large area in contact with the reservoirs: whether one can safely assume that there is a single tunneling point contact is therefore extremely doubtful. It is then perfectly conceivable that other boundary conditions than $\begin{pmatrix} eV_S \\ eV_D \end{pmatrix} = \begin{pmatrix} \mu_+^0 \\ \mu_-^0 \end{pmatrix}$ may be valid, leading therefore to a conductance $G \neq G_0$.

The quantum wire or individual single wall carbon nanotubes experiments finding conductances unquantized at G_0 may therefore be explainable using interacting electrons within our formalism.

The variety of boundary conditions reflects simply the nature of the equilibrium achieved between the reservoirs and the LL: the chemical potentials of the charge carriers within the LL (i.e. the chiral chemical potentials) need not be identical with those of the charge reservoirs. Only for symmetric coupling of the electrodes and if $a^S = 1$ does one find that $eV_S = \mu_+$ and $eV_D = \mu_-$: Kane and Fisher calculations fall into that class. That boundary condition is natural for the FQHE because it is sensible for the chemical potentials of the reservoirs to be in equilibrium with those of the charge carriers, since the contact region between the FQHE condensate and the reservoirs is large. However as shown microscopically by Chamon et al.¹⁷, if the contact with the reservoirs is not perfect (e.g. a granularity limits the number of tunneling points), such an equilibrium may not be achieved.

V. TWO-TERMINAL GEOMETRY WITH AN IMPURITY.

A. New boundary conditions.

We now insert a weak local impurity in the wire:

$$V = u\delta(x)(\Psi_R^+\Psi_L + h.c.). \quad (44)$$

A current will therefore be backscattered and the potentials need not be identical across the impurity. We use eq.(36) to write boundary conditions in the presence of an impurity:

$$\begin{aligned} eV_S - \bar{\mu}_L &= R_S I e \\ \bar{\mu}_R - eV_D &= R_D I e \end{aligned} \quad (45)$$

where the index R/L refers to $\mu_{R/L}^\pm$ are the chiral chemical potentials to the right or the left of the impurity and $\bar{\mu}$ is the average between the chiral chemical potentials. To have conditions on the sole chemical potentials it suffices then to remark that $I = \frac{K e}{h} (\mu_L^+ - \mu_L^-) = \frac{K e}{h} (\mu_R^+ - \mu_R^-)$, so that:

$$\begin{aligned} eV_S - \bar{\mu}_L &= R_S G_K (\mu_L^+ - \mu_L^-) \\ \bar{\mu}_R - eV_D &= R_D G_K (\mu_R^+ - \mu_R^-) \end{aligned} \quad (46)$$

In spite of these linear relations the chiral chemical potentials need not depend linearly on the external voltages (there are four chemical potentials for two linear relations). There is also a non-linear contribution due to the backscattering at the impurity.

B. What is the backscattering current?

In the presence of the impurity, some of the current is backscattered. It is usually assumed that the backscattering current is simply the difference between the current in the absence of impurity and the current in the presence of the impurity. This is only correct for non-interacting systems, but for a LL this will depend on both the conductance G_{2t} and K . We must go back to the definition of the backscattering current as the velocity times the density difference of right-movers on the left and on the right of the impurity:

$$i_B = u e (\rho_L^+ - \rho_R^+). \quad (47)$$

This is also equal by charge conservation to: $u e (\rho_L^- - \rho_R^-)$ with obvious notations. We can also relate i_B to the chiral chemical potentials by using the fact that $u \rho_L^+ = \frac{K}{h} \mu_L^+$ (with similar relations for the $-$ chirality and for the potentials to the right of the impurity). Therefore:

$$i_B = \frac{K e}{h} (\mu_L^+ - \mu_R^+) = \frac{K e}{h} (\mu_L^- - \mu_R^-). \quad (48)$$

Using the new boundary conditions (46):

$$\begin{aligned} eV_S - eV_D &= \bar{\mu}_R + R_D I e + \bar{\mu}_L + R_S I e \\ &= R_K i_B e + (R_S + R_D) I e \end{aligned}$$

where $R_K = h/Ke^2$. This can be recast as:

$$\begin{aligned} I &= I_0 - \frac{R_K}{R_S + R_D} i_B \\ &\neq I_0 - i_B \end{aligned} \quad (49)$$

where

$$I_0 = \frac{eV_S - eV_D}{e(R_S + R_D)} \quad (50)$$

is the current in the absence of an impurity and is therefore also the saturation current, i.e. the maximal current which can be reached when one goes to large voltages. The fact that $i_B \neq I_0 - I$ contrary to the naive expectation stems from the contact resistances: the difference between i_B and $I - I_0$ is akin to the difference between a two-terminal and a four-terminal measurement. $I_0 - I$ takes into account the resistance at the contacts while i_B is more intrinsic and measures the net current which is backscattered *locally* at the impurity.

VI. SHOT NOISE IN A TWO-TERMINAL GEOMETRY.

What are the elementary excitations of the LL? The textbook answer is that there are two kinds of excitations: (1) bosonic density fluctuations (plasmons); (2) zero modes ladder operators which change the number

of particles at each Fermi point but have no dynamics³. It is seldom remarked that such a description of the excitations found for the LL through the bosonization method is also valid for free electrons. What this means is that for free electrons there are two equivalent manners of describing the elementary excitations (corresponding to two basis of eigenstates): (1) the usual manner, in terms of charged quasiparticles (the electron and the hole); (2) and the one provided by bosonization, which yields bosonic density fluctuations and ladder operators. The two descriptions differ markedly in that the second involves charged excitations which have no dispersion, while in the first the charge dynamics is described by the usual quasiparticles.

For the LL it can be shown that exactly in the same manner there exists a basis of charged quasiparticles. However instead of the Landau quasiparticle one finds fractional elementary excitations, which may even carry irrational charges. In particular the particle-hole continuum of Fermi liquid theory is replaced by a quasiparticle-quasihole continuum of excitations which are the analogs of Laughlin quasiparticles¹⁶. For the chiral LL (the edge states of the FQHE) they have been detected through shot noise. *In the case of the non-chiral LL a marked difference is that such shot noise experiments would allow to detect irrational charges* (the FQHE filling fraction ν which is a rational number is replaced by the LL parameter K).

Present theories of shot noise can be roughly separated into two camps: **A** Kane, Fisher, Balents et al.^{22,23} predict a Fano factor equals to Ke . This is commonly interpreted as the proof that excitations of charge Ke are responsible for the noise. This calculation however makes no explicit modelization of the reservoirs; **B** Ponomarenko et al., Egger et al.²⁴ work with the inhomogeneous LL (two terminal geometry which models the reservoirs as 1D Fermi liquids on a half-line) and find a Fano factor or excitations of charge equals to e . We note that Blanter and Büttiker have argued against this last result by noting that the shot noise should not depend on the reservoirs since this is a measure of the charge backscattered *locally* by the impurity. We discuss now these two sets of theories: **A** we apply our boundary conditions formalism to the shot noise theory of Kane and Fisher; **B** for the inhomogeneous LL we discuss the meaning of their result in the light of the identities derived in the previous section.

A. Kane-Fisher approach

The shot noise through a weak impurity in a LL was first computed by Kane and Fisher (for the edge states of the FQHE and before the actual proof that there exists also Laughlin quasiparticles in the LL) by using the Keldysh formalism applied to an effective lagrangian found by integrating out the degrees of freedom away

from an unique impurity (ϕ is the standard LL phase field at the location of the impurity)²²:

$$L = \frac{1}{2K} \sum_n |\omega_n| |\Phi(i\omega_n)|^2 + \int d\tau v \cos(2\sqrt{\pi}\phi(\tau)). \quad (51)$$

Although initially intended for the edge states of the FQHE the calculation is also valid for the non-chiral LL. Kane and Fisher find that the current and the noise are given respectively by:

$$I = K \frac{e^2}{h} \left(\frac{da}{dt} - V \right), \quad (52)$$

$$S_I = Ke \left(K \frac{e^2}{h} \frac{da}{dt} - I \right) \quad (53)$$

where $V = \langle \frac{h}{e} v \sin(2\sqrt{\pi}(\phi + Ka)) \rangle$, with a source term $\int j \cdot a$ added to the lagrangian. Kane and Fisher assumed that

$$e \frac{da}{dt} = e(V_S - V_D). \quad (54)$$

In the absence of impurity $V = 0$, eq.(52) leads to

$$I_0 = K \frac{e^2}{h} \frac{da}{dt}, \quad (55)$$

on the other hand according to eq.(8), $I_0 = K \frac{e}{h} (\mu_+ - \mu_-)$, together with eq. (54) this then implies that $e(V_S - V_D) = (\mu_+ - \mu_-)$. Therefore within Kane-Fisher approach and assuming eq. (54) the “two terminal” conductance value is $G_{2t} = Ke^2/h$ in the absence of an impurity. Moreover as discussed in **section 4.4** (assuming symmetric coupling to source and drain) this calculation falls into the class of boundary conditions which correspond to $\begin{pmatrix} \mu_+ \\ \mu_- \end{pmatrix} = \begin{pmatrix} eV_S \\ eV_D \end{pmatrix}$, i.e. equilibrium of the reservoirs chemical potentials with those of the LL. In order to obtain values of the conductance different from Ke^2/h , it is sufficient to change the previous assumption eq. (54): other classes of boundary conditions are found simply by *assuming that the response of the LL is totally driven by the values of the chiral chemical potentials (the chemical potentials of the charge carriers of the LL) in the absence of an impurity*, and not by the reservoirs potentials (since there is no reason why they should be equal). We therefore modify eq.(54) into:

$$e \frac{da}{dt} = \mu_+ - \mu_- \quad (56)$$

$$= \frac{R_K}{R_S + R_D} e(V_S - V_D) \quad (57)$$

where in the second line the boundary conditions (35) in the absence of impurity have been used. Eq. (57) together with eq. (55) then lead to the value of the “two-terminal” conductance that we obtained with our

approach in **section 4** for the same boundary conditions, namely $G_{2t} = 1/(R_S + R_D)$.

So far we have shown that by assuming the source term definition Eq. (57) instead of Eq. (54) it is possible to adapt Kane-Fisher calculations to reproduce the various boundary conditions in the absence of impurity.

We can now reconsider Kane and Fisher’s calculations for the shot noise, i.e. in the presence of an impurity in the bulk of the LL. More precisely we express now the shot noise as a function of either $I_0 - I$ (the deviation to the saturation current) or as a function of the backscattering current i_B , for the various boundary conditions. According to eq. (55) and eq. (53) we always have *independently of the boundary condition chosen*:

$$S_I = Ke(I_0 - I). \quad (58)$$

Therefore, using eq. (49) $I = I_0 - \frac{R_K}{R_S + R_D} i_B = I_0 - \frac{G_{2t}}{G_K} i_B$ the last equality is also recast as:

$$S_I = \frac{G_{2t}}{G_0} e i_B, \quad (59)$$

where G_{2t} is the two terminal conductance that reflects the contact resistances in the absence of the impurity. The shot noise Fano factor might therefore appear to depend on whether one refers to the backscattering current i_B or to $I_0 - I$, the deviation to the saturation current. But since S_I is the fluctuation of the current I , the physical shot noise charge must be measured with respect to the current I and not with respect to i_B . The shot noise charge is therefore $\frac{S_I}{I_0 - I} = Ke$, independently of the boundary condition realized in the system and is not equal to $\frac{S_I}{i_B}$.

At any rate what is directly measured is always I or I_0 : i_B is only indirectly accessible through for instance eq. (49).

In summary, within the Kane and Fisher calculation by the Keldysh method, it is therefore possible to have (i) an ohmic conductance distinct from Ke^2/h and (ii) a shot noise charge equal to Ke independently on the value of the ohmic conductance.

B. Inhomogeneous model approach

The role of the reservoirs on the measure of the shot noise of a LL was examined in two papers²⁴, which make calculations on the inhomogeneous LL model for which the boundary condition is $\begin{pmatrix} eV_S \\ eV_D \end{pmatrix} = Q[K] \begin{pmatrix} \mu_+ \\ \mu_- \end{pmatrix}$ as discussed in section 4.4. They both find that $S_I = e(I_0 - I)$ with a conductance $G_{2t} = G_0$. It is interesting to remark that the equation $S_I = e(I_0 - I)$ can be recasted in terms of the backscattering current noise as:

$$S_{i_B} = Ke i_B,$$

since $I_0 - I = i_B/K$ when $G_{2t} = G_0$.

The result of Ponomarenko et al. and Egger et al.²⁴, acquires then the following interpretation: the charge Ke is not found in the shot noise for the total current in the LL because it is really the correlations of the backscattering current which should be measured. Since the impurity backscatters charge Ke Laughlin quasiparticles, the backscattering current correlations must contain the information on the charge backscattered by the impurity.

This is in disagreement with Kane and Fisher theory even when this last theory is modified by our boundary conditions formalism in order to reproduce the situation $G_{2t} = G_0$. We are unable as yet to explain the discrepancy between the two approaches.

Lastly, we note that taking the relation $S_{i_B} = Ke i_B$ as a starting point, this then according to our formalism leads inevitably to $S_I = \frac{G_{2t}}{G_0} e (I_0 - I)$. An experimental test of this last suggestion would require an independent measurement of K and i_B . In FQHE such independent measurement is possible because the two chiralities of the effective LL are physically well separated and a direct measure of K is then possible through the Hall conductance. In contrast to the FQHE, i_B is not experimentally measurable in Carbon nanotubes: this then means in turn that even though $S_{i_B} = Ke i_B$ the charge of Laughlin quasiparticles in a non-chiral LL is not directly measurable through shot noise in a two-terminal geometry.

Shot noise experiments will hopefully settle the issue. In this respect some experiments on carbon nanotubes are in progress²⁶.

VII. CONCLUSIONS.

We proposed in this paper a new formalism which models the joint system LL+electrodes as a single LL with no electrodes but subjected to boundary conditions on its chiral chemical potentials. We were able to show in a solvable toy-model that such boundary conditions can indeed be derived explicitly. That model is quite remarkable because the conductance matrix of the LL in contact with an arbitrary number of terminals can be computed; it is found that the probabilistic scattering approach fails: it would lead to negative probabilities for the transmission of electrons. The obvious advantage of our formalism is that it avoids discussion of the detailed microscopics of a system, but yields a classification of the joint system LL+electrodes and then makes precise predictions for the transport. In particular the Landauer-Büttiker view of the contact resistance as resulting from a mismatch between chemical potentials is recovered; if the charge backscattered by an impurity in a LL is $\frac{S_{i_B}}{i_B}$ we find that the shot noise of the total current does not allow a measure of the fractional charge Ke of Laughlin particles in a LL.

It is easy to generalize our formalism to the case of several channels: (i) several conducting channels as in

carbon nanotubes; (ii) spin transport: this arises with ferromagnetic reservoirs; (iii) application of a magnetic field on the LL, which breaks the spin-charge separation.

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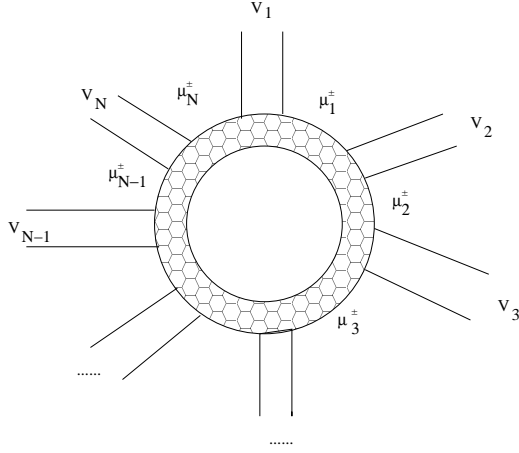


FIG. 1. LL connected to many electrodes in a loop geometry. V_n is the potential of electrode n . μ_n^\pm are the chiral chemical potential on the left of electrode n . Due to current injection at each terminal the chiral chemical potentials may change their values across them: accordingly μ_n^\pm on the left of terminal n is changed to μ_{n+1}^\pm on the right of the same terminal.

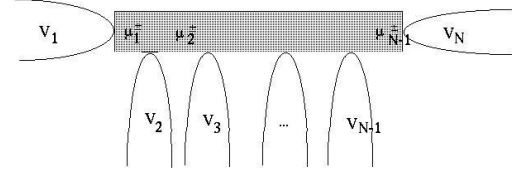


FIG. 2. Same as Fig. 1 but for LL connected to many electrodes in a rod geometry.

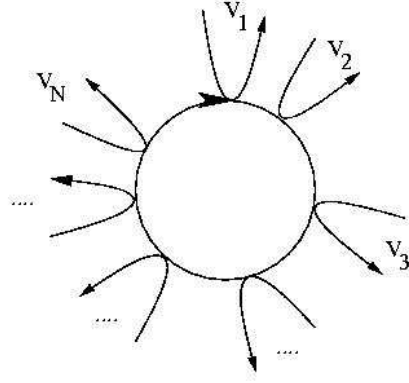


FIG. 3. Model of a chiral LL connected to many chiral electrodes in a loop geometry.

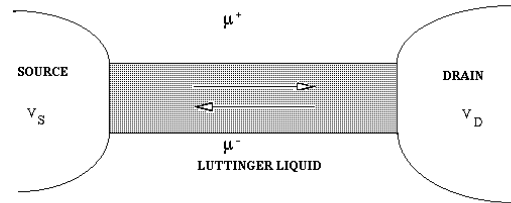


FIG. 4. LL in a two terminals geometry.